

Forecasting of Monthly and Annual Rainfall at Rajasthan Region

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Abstract—Rainfall is a phenomenon, which directly or indirectly affects all the sectors like agriculture, insurance, industry and other allied fields. It is well established that the rainfall is changing on both the global and the regional scales due to global warming. So the prediction of actual amount of rainfall is very complex till now. But now a day's stochastic rainfall models are widely used in hydrologic analysis for rainfall forecasting. These models can provide precipitation input to models whenever data are not available. In this paper, Markov first order process is used to calculate the amount of rainfall for all the districts of Rajasthan. For this purpose rainfall data for 41 years (1970–2010) for each 33 districts of Rajasthan is used. Using first order Markov chain model monthly and yearly magnitude of rainfall is predicted for the year 2011. All the parameters of this model were predicted from the historical rainfall records. Obtained results are compared with the observed rainfall. This model has 18.89% average relative error for all 33 districts of Rajasthan for monthly analysis of annual rainfall where as 24.02% is the observed for annual rainfall. It is observed that monthly total of rainfall for each district is more efficient than yearly predicted rainfall.

Keywords: Stochastic, hydrologic analysis, Markov first order, rainfall.

1. INTRODUCTION

Rajasthan contributes 10.4 % of India's total area. Geographic features of Rajasthan are composed of Thar desert and the Aravalli mountain ranges, which runs almost from one end to another (more than 850 km). Although a large percentage of the total area is desert with little forest cover. As the population of Rajasthan is increasing day by day, ranked 7th populous state of India. People are directly or indirectly engaged in wide range of activities such as preserving water for further use. Rainfall is the most important natural phenomena which decide the stability of population in a region. Directly or indirectly all human beings are dependent on rainfall. So, knowing an idea about the status of rainfall in advance is very useful for farmers as well designers to plan and design the systems related to rainfall activities.

The development of a rainfall model is increasingly in demand, not only for data-generation purposes, but also to

provide some useful information in various applications, including water resource management and the hydrological and agricultural sectors [1]. There are four techniques used in rainfall prediction: Statistical technique, Stochastic method, Artificial Neural Network and Numerical weather prediction [2]. Stochastic rainfall generators are widely used in hydrologic analysis because they can provide precipitation input to models in situations in which data are not available. Rather than attempting to reproduce actual rainfall records, stochastic models aim at generating synthetic precipitation time series, the statistics of which match those of the observed series [3].

A first order Markov chain only require one variable (like temperature, rainfall, fog, frost, cloudiness, wind) to forecast its component at some later time. The Markov chain models have 2 advantages:

- The forecasts are available immediately after the observations
- They require minimal computations after the climatologically data have been processed.

Since Gabriel & Neumann [4] proposed that the sequence of wet and dry days be represented by a Markov chain, the latter has been one of the most widely and commonly used models for representing daily precipitation occurrence processes [5][6][7] analyzing daily precipitation records for 25 years at more than 100 stations in the United States observed that the order of conditional dependence of daily precipitation occurrences depends on the season and geographical location. Wang and Nathan [8] proposed a daily monthly mixed algorithm to preserve the monthly rainfall characteristics explicitly. In this model, two daily rainfall sequences are generated using daily and monthly parameters and the daily rainfall sequences generated from the daily parameters are adjusted using the other sequence generated from the monthly parameters. This adjustment ensures that the monthly characteristics are preserved in the generated daily rainfall sequences. However, the model fails to preserve the annual rainfall characteristics. Thyer and Kuczera[9] developed a

Table 1: Summary statistics of rainfall for Rajasthan region

Station no.	District	Latitude	Longitude	Min	Max	Average
1	Ajmer	26° 27' N	74° 37' E	205.3	1163	554.8
2	Alwar	27° 34' N	76° 36' E	223.8	1445.1	640.08
3	Banswara	23° 33' N	74° 27' E	552	2591	1093.96
4	Baran	25° 07' N	76° 31' E	254	1537.4	816
5	Barmer	25° 45' N	71° 24' E	28.3	759	275.74
6	Bharatpur	27° 13' N	77° 30' E	218.5	1041	628.88
7	Bhilwara	25° 44' N	74° 20' E	307	1120	682.1
8	Bikaner	28° 01' N	73° 18' E	32	580	271.48
9	Bundi	25° 27' N	75° 38' E	275.5	1245.4	709.91
10	Chittorgarh	24° 53' N	74° 38' E	470.7	1330.2	801.91
11	Churu	28° 18' N	74° 58' E	165	5554	520.16
12	Dausa	26° 54' N	76° 20' E	300	1363.2	671.17
13	Dholpur	26° 42' N	77° 53' E	171.8	1046	641.24
14	Dungarpur	23° 51' N	73° 43' E	319	1611	746.47
15	Ganganagar	29° 55' N	73° 53' E	98	752	294.85
16	Hanumangarh	29° 35' N	74° 19' E	75.5	547	254.51
17	Jaipur	26° 55' N	76° 50' E	222.4	1036	616.26
18	Jaisalmer	26° 55' N	70° 55' E	30.5	527	203.31
19	Jalore	25° 21' N	72° 37' E	133.2	1047.4	436.4
20	Jhalawar	24° 36' N	76° 09' E	532.2	1614.5	909.44
21	Jhunjunu	28° 08' N	75° 23' E	123	828	391.18
22	Jodhpur	26° 18' N	73° 02' E	91	815	372.39
23	Karauli	26° 30' N	76° 01' E	282	1329.4	780.38
24	Kota	24° 53' N	75° 57' E	379.7	6666	836.75
25	Naguar	27° 12' N	74° 45' E	110	1259	417.43
26	Pali	25° 47' N	73° 20' E	90	1022.4	415
27	Pratapgarh	24° 03' N	74° 47' E	467	1668.7	923.71
28	Rajsamand	24° 04' N	73° 53' E	214	1039	548.98
29	Sawai	25° 59' N	76° 22' E	295	1244.2	784.11
30	Sikar	27° 37' N	75° 09' E	124	1012.3	510.71
31	Sirohi	24° 53' N	72° 52' E	190.9	1577.6	613.16
32	Tonk	26° 10' N	75° 48' E	221.3	1078	623.65
33	Udaipur	24° 35' N	73° 42' E	215.1	1095.5	569.63

3. METHODOLOGY

For all the districts of Rajasthan monthly and yearly rainfall data is collected. Depending upon the minimum and maximum value of rainfall frequency distribution table is formed and rainfall data of each district is divided into seven states. Using first order Markov chain model transitional probability matrices are formed showing the transition of rainfall from one state to another, then for next year transitional probability matrix is formed. All the parameters of this model were predicted from the historical rainfall records.

4. MARKOV CHAIN MODELLING

A Russian mathematician, Markov, introduced the concept of a process (later named after him 'a Markov process') in which a sequence or chain of discrete states in time for which the probability of transition from one state to any given state in the next step in the chain depends on the condition during the previous step [11]. Gabriel and Neumann [4] analyzed the occurrence of rain by fitting a two-state, first-order Markov chain. Carey and Haan [12] used multi-state Markov chain models to generate daily rainfall depths.

A first order Markov chain is a stochastic process having the property that the value of the process at time t , X_t , depends only on its value at time $t-1$, X_{t-1} , and not on the sequence of values that the process passed through in arriving at X_{t-1} .

In general, the number of states at each time instant assumed as n . Hence, there will be $n \times n$ transitions between two successive time instances. It is then possible to find the number of transition probabilities, p_{ij} from a state at time t to another state at time $t+1$, and accordingly, the following transition probability matrix, $P_{t,t+1}$ can be prepared from observed rainfall data. The structure of the transition probability matrix would be

$$P = P_{t,t+1} = \begin{pmatrix} P_{11} & P_{12} & P_{13} \dots P_{1c} \\ P_{21} & P_{22} & P_{23} \dots P_{2c} \\ \vdots & \vdots & \vdots \\ P_{c1} & P_{c2} & P_{c3} \dots P_{cc} \end{pmatrix} \quad (1)$$

And it can be predicted by equation no. 2 [10]:

$$P_{ij} = \frac{f_{ij}}{\sum_{j=1}^c f_{ij}} m \quad (2)$$

Where f_{ij} = historical frequency of transition from state i to state j and

C = the maximum number of state

With the rainfall data this matrix shows the transition probabilities, p_{ij} of rainfall in state i at time t to state j at time $t+1$ given n rainfall state the following properties of the transition matrix are valid by definition. Any state probability varies between zero and one. Rotationally,

$$0 < p_{ij} < 1.0 \text{ where } i, j = 1, 2, \dots, C$$

$$\sum_{j=1}^n P_{ij} = 1 \quad (3)$$

Where $i = 1, 2, \dots, C$

Once P is known, all that is required to determine the probabilistic behaviour of the Markov Chain is the initial state of the chain. In the following, $p_j^{(n)}$ denotes the probability that the chain is in state j at step or time n . The $1 \times C$ vector $P^{(n)}$ has elements $P_j^{(n)}$

Thus

$$p^{(n)} = [p_1^{(n)}, p_2^{(n)}, \dots, p_c^{(n)}]$$

$$p^{(1)} = p^{(0)} p \quad (4)$$

Where, $p^{(0)}$ is the initial probability vector.

In general,

$$p^{(n+c)} = p^{(c)} p^{(n)} \quad (5)$$

Where, p^n is the n^{th} power of p

In this study,

$$P^{(0)} = [1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7] \quad (6)$$

5. RESULTS AND DISCUSSION

For monthly analysis of rainfall using the Markov chain model, we consider the seven states and boundaries for each district is different depending upon the minimum and maximum value of rainfall for each district. Table 3 shows the frequency distribution table for Pali district for the month of August.

Table 3: Frequency distribution of rainfall events

State	Limits
A	0.0 – 76.28
B	76.28 - 152.57
C	152.57 - 228.857
D	228.57 - 305.14
E	305.14 - 381.42
F	381.42 - 457.71
G	457.71 – 534

Here the state-A indicated that, there is rainfall magnitude varies from 0 to 76.28 mm. Since the transition probabilities are calculated month wise has the order 7 X 7. For the month of august the transition probabilities given below:

$P_{ij} =$

$$\begin{pmatrix} 0.214 & 0.285 & 0.214 & 0.0714 & 0 & 0.142 & 0.071 \\ 0.384 & 0.384 & 0.076 & 0.076 & 0 & 0 & 0.076 \\ 0.428 & 0.428 & 0.142 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 \end{pmatrix}$$

In the same way the monthly transition probability matrix for each district for each month is calculated. After the calculating the transition probability matrix for each month for each year we calculate the probability matrix for next year month wise using equation 4. Finally sum up the rainfall magnitude for each district to get the annual rainfall.

Comparison of historic and predicted rainfall for month of august is presented in Figure 3 and table 4. For the month of January, March, April, May, June, October, November and December the predicted rainfall is more than the observed rainfall. It could be due to natural phenomenon of rainfall. As rainfall depends on many factors such as wind, temperature, humidity, geographical features of region etc. But in this model we consider only one variable that is past record of rainfall as compare to many other factors. For 41 years it has been observed that every month must have some amount of rainfall in any of the successive years. Due to this model will never give zero amount of rainfall. When daily analysis of rainfall is done for monsoon months then there is still certain amount of rainfall is predicted on each day. But in actual all days are not rainy in monsoon month. So this model does not holds good for daily analysis of rainfall as compared to monthly and yearly forecasting.

Table 4: Statistical comparison of predicted rainfall and observed rainfall

Observed rainfall (mm)	0	6	0	0	7	32	202	192	82	0	0	0
Predicted rainfall (mm)	2.6	5.3	3.6	3.4	14.4	41.5	177.9	173.5	55	30	7.3	1.8
Absolute Error (mm)	2.6	0.7	3.6	3.4	7.4	9.5	24.1	18.5	27	30	7.3	1.8
MAD = 11.31 MSE = 232.02 RMSE = 15.24 RSE = 0.05 RAE = 0.23												

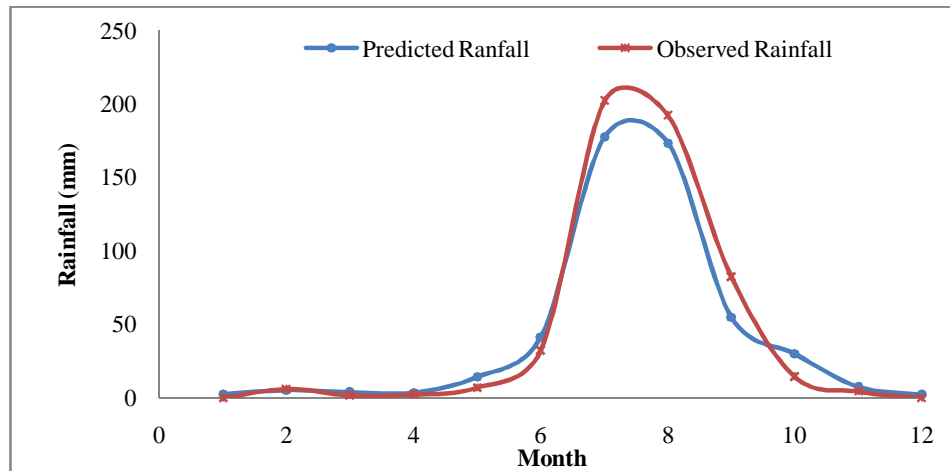


Figure 3: Variation of observed rainfall for Pali district with the predicted rainfall.

In the similar way monthly rainfall for the year 2011 is predicted for all the districts of Rajasthan shown in table 5. Comparison between observed and predicted rainfall for Alwar and Jodhpur district is shown in Figs. 4 and 5.

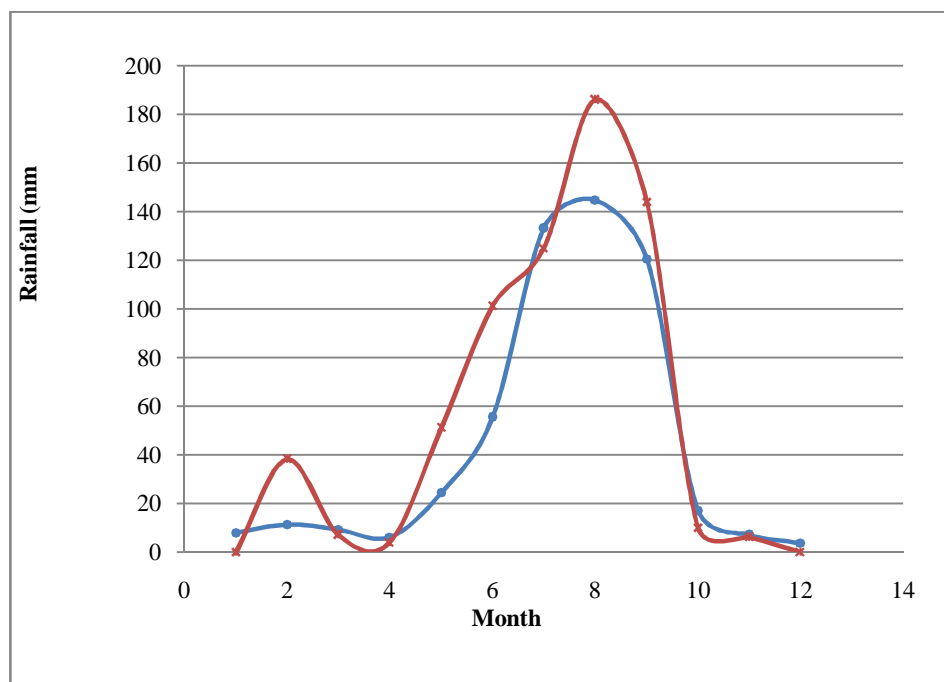


Figure 4: Comparison of observed and predicted rainfall for Alwar.

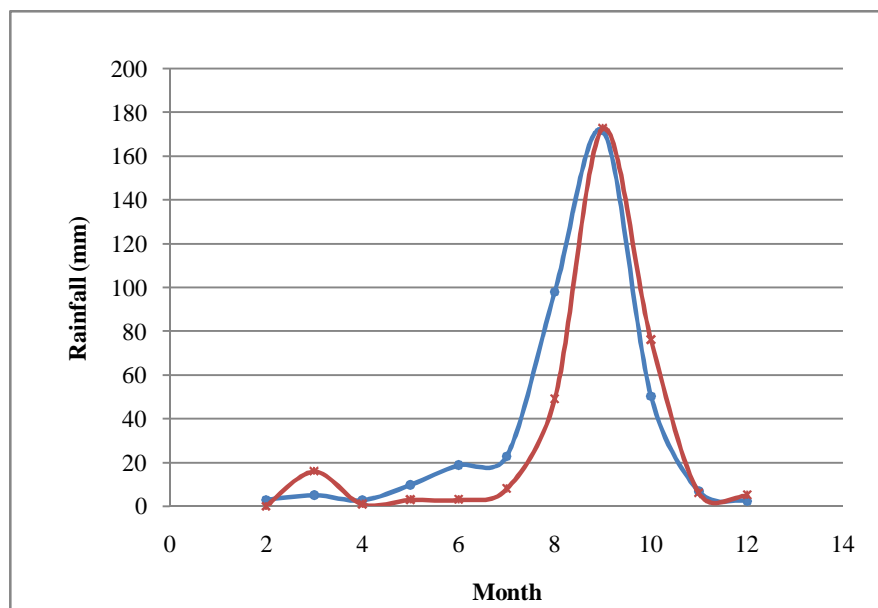


Figure 5: Comparison of observed and predicted rainfall for Jodhpur

On applying the regression analysis to the observed data and predicted data. It has been observed that for linear variation the coefficient of correlation r lies between 0.557 to 0.987 as shown in Figure 8 but district Ganganagar is having very low value of r 0.369. Many districts such as Pali, Pratapgarh, Nagaur, Jhunjunu, Chittorgarh, Jodhpur, Bhilwara, Bundi and Dungarpur are having value of r more than 0.9 which indicates a strong strength and the direction of a linear relationship between observed data and predicted data. Churu, Jaisalmer, Kota, Bharatpur, Karauli, Bikaner, Barmer and Ganganagar have the value of r less than 7.

When the monsoon months of next successive years are considered, the value of coefficient of determination r^2 for Pali and Jodhpur districts of Rajasthan as shown in Figs. 6 and 7.. The coefficient of determination, r^2 , is useful because it gives the proportion of the variance (fluctuation) of predicted rainfall that is predictable from the observed rainfall. It is a measure that

allows us to determine how certain one can be in making predictions from a certain model/graph. Strong relationship exist between the predicted and observed value. These regions like Pali and Jodhpur have uniform rainfall patterns which increase the efficiency of model.

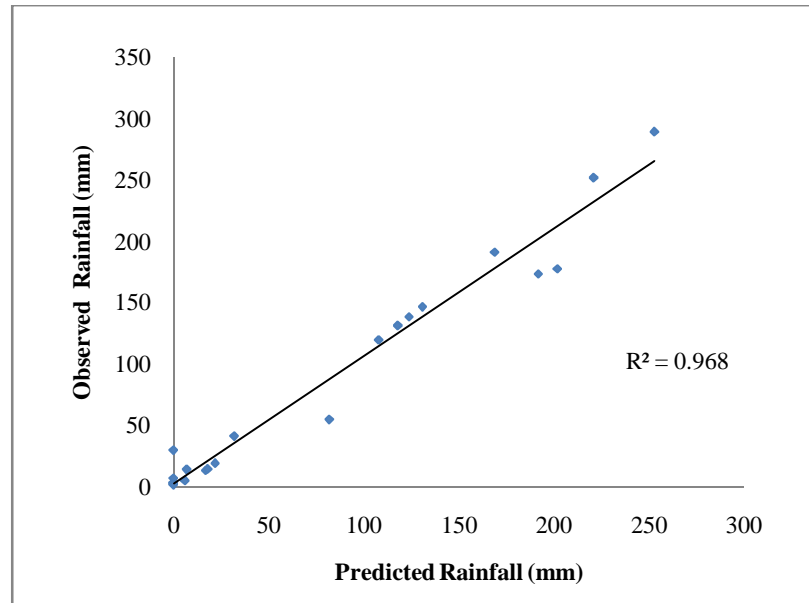


Figure 6: Comparison of observed and predicted rainfall for Pali.

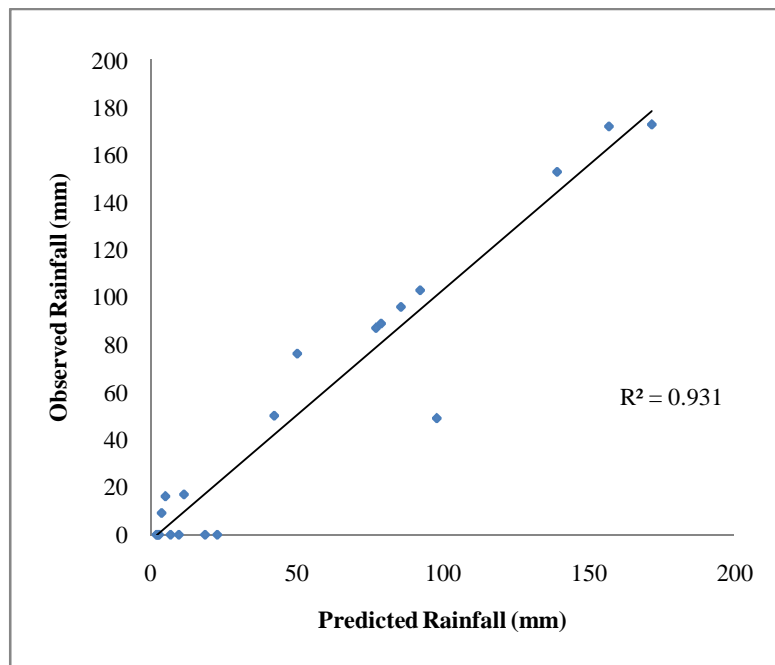


Figure 7: Comparison of observed and predicted rainfall for Jodhpur

rainfall that is predictable from the observed rainfall. It is a measure that allows us to determine how certain one can be in making predictions from a certain model/graph. Strong relationship exist between the predicted and observed value. These regions like Pali and Jodhpur have uniform rainfall patterns which increase the efficiency of model.

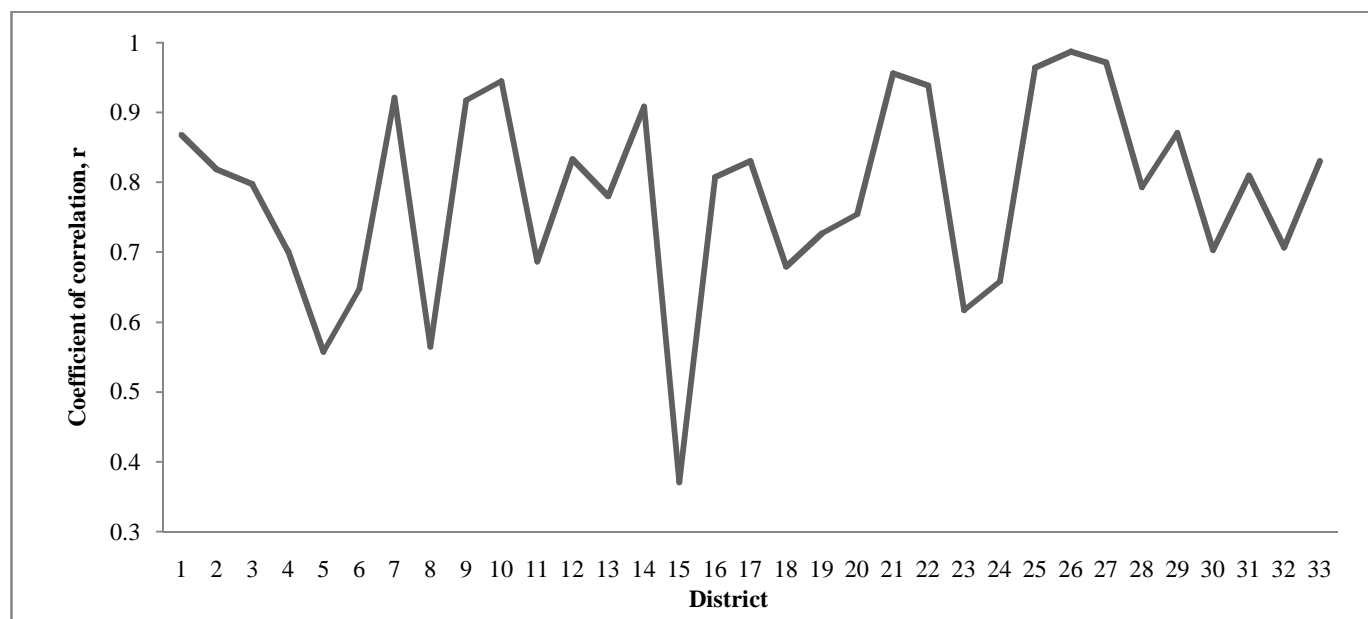


Figure 8: Value of coefficient of correlation (r) for each district of Rajasthan

Depending upon maximum and minimum value of rainfall for the period of 41 year frequency distribution table is made for each district of Rajasthan for yearly analysis. Using Markov first order process, annual rainfall for the year 2011 is predicted. Table 5 shows annual rainfall for each district of Rajasthan.

Table 5: Annual rainfall by monthly total and yearly prediction method

Station no	District	Monthly predicted	Yearly predicted	Annual observed	Relative percentage error for monthly prediction	Relative percentage error for yearly prediction
1	Ajmer	562.5622	632.7628	491.6	14.43495	28.71496
2	Alwar	749.35	647.9745	813	7.829028	20.29834
3	Banswara	1303.721	1048.673	1382	5.664182	24.11918
4	Baran	895.2344	942.2283	1507	40.59493	37.47656
5	Barmer	358.9122	303.8073	343	4.639125	11.42645
6	Bharatpur	723.4365	684.6338	743	2.633042	7.855478
7	Bhilwara	831.6838	735.7442	730	13.92929	0.786877
8	Bikaner	368.9256	320.0977	549	32.80044	41.6944
9	Bundi	791.7526	741.1309	750	5.567013	1.182552
10	Chittorgarh	962.539	824.9176	817.5	17.74177	0.907352
11	Churu	483.7962	428.7479	648	25.34009	33.8352
12	Dausa	835.3287	747.8881	918	9.005588	18.53071
13	Dholpur	727.8471	694.3096	746	2.433365	6.929008
14	Dungarpur	834.7905	853.9054	977	14.55573	12.59924
15	Ganganagar	352.0045	311.5805	453	22.29481	31.21843
16	Hanumangarh	340.7022	294.8253	297	14.71455	0.732222
17	Jaipur	677.1532	560.9456	798.6	15.20746	29.75888
18	Jaisalmer	284.7934	214.2483	277	2.813502	22.65404
19	Jalore	602.0266	480.896	582.4	3.369952	17.42857
20	Jhalawar	1007.58	1420	981.0295	2.706392	44.7459
21	Jhunjunu	540.6383	420.9323	646	16.30986	34.8402
22	Jodhpur	392.6108	394.6796	314	25.03529	25.69414
23	Karauli	895.9629	810.056	500	79.19258	62.0112
24	Kota	865.0292	668.1989	968	10.63748	30.97119
25	Naguar	476.3722	462.6321	230	107.1183	101.1444

26	Pali	515.7331	473.002	521	1.010921	9.212668
27	Pratapgarh	1015.418	973.1427	1381	26.47227	29.53348
28	Rajsamand	631.5316	577.2375	522	20.98307	10.5819
29	Sawai	973.6401	824.0775	946	2.921786	12.88821
30	Sikar	653.9681	606.8124	428.4	52.65362	41.64622
31	Sirohi	758.3426	769.1465	751.4	0.923955	2.361791
32	Tonk	686.6529	670.1088	738	6.957602	9.19935
33	Udaipur	677.1532	560.9456	798.6	15.20746	29.75888

For the year 2011, monthly and annual rainfall is calculated using the Markov First order process. It has been observed that sum of monthly predicted rainfall is more accurate as compared to annually calculated. Figure 9 shows the variation of observed rainfall, sum of monthly predicted rainfall and yearly predicted rainfall.

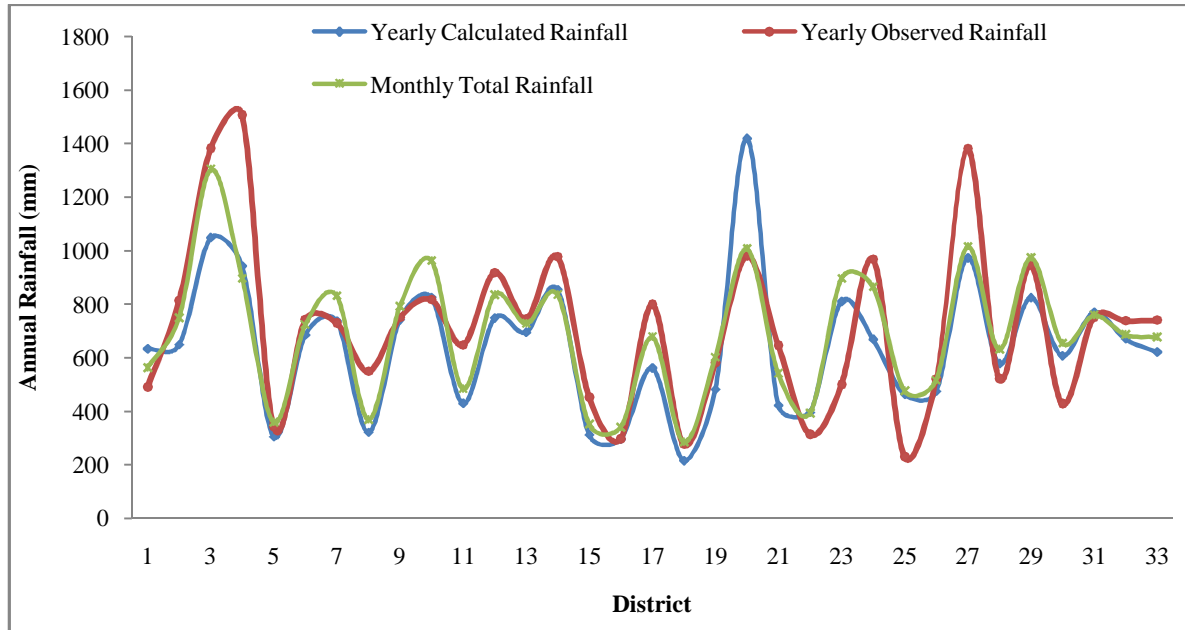


Figure 9: Variation of observed rainfall, sum of monthly predicted rainfall and yearly Predicted rainfall

This model has 18.89% average relative error for all 33 districts of Rajasthan for monthly analysis of annual rainfall where as 24.02% is the observed for annual rainfall. So monthly modelling is more accurate as compared to yearly modelling. Figure 10 shows the relative error corresponding to monthly total and yearly basis. It has been observed that Alwar, banswara, Barmer, Bharatpur, Bundi, Dholpur, Jaisalmer, Jalore, Jhalawar, Pali, Sawaimadhupur, Sirohi and tonk shows less than 10% relative error for monthly total rainfalls.

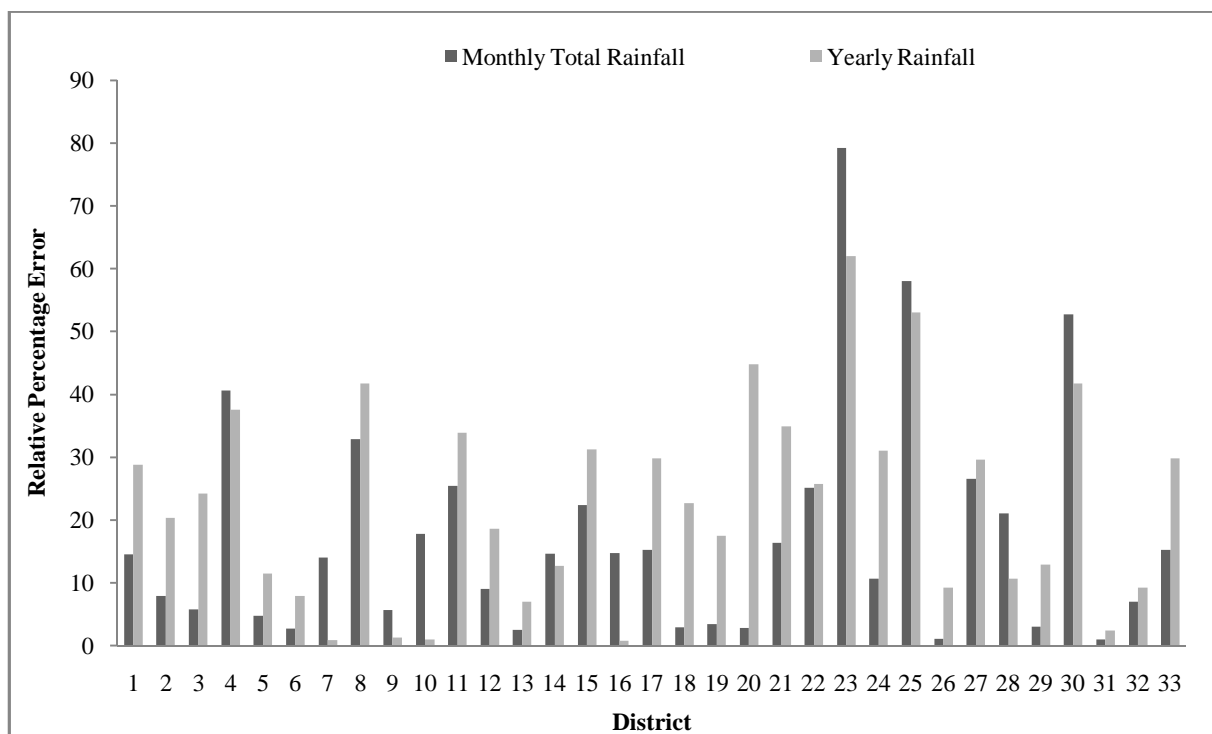


Figure 10: Relative percentage error of 33 districts of Rajasthan

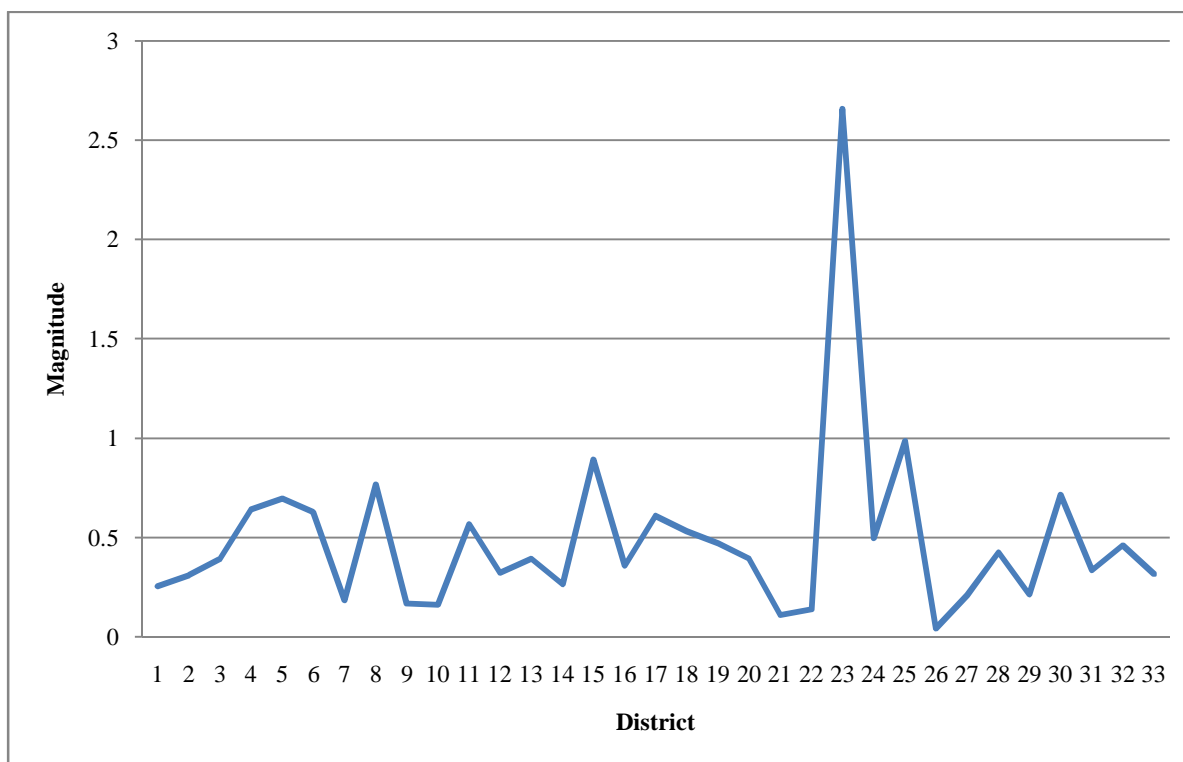


Figure 11: Variation of relative standard error (RSE) for all district of Rajasthan

It is observed that the RSE (relative standard error) is less than 1 for all the district of Rajasthan except Karauli. Karauli has 2.6 RSE which is very large as compared to other districts error. RSE is an absolute gauge between the observed results and the

predicted results. If the standard error is large relative to the results; large relative standard errors suggest the results are not significant. But in Figure 11 it is observed almost all the districts are having very low RSE so this modelling is very effective for prediction of amount of rainfall on monthly and yearly basis.

When we compare regression analysis for the observe data of rainfall with monthly predicted total in Figure 12 and with annually rainfall in Figure 13. It has been observe that value of coefficient of correlation r for Figure 12 is 0.83 which shows a strong connection between the observed data and predicted data. For Figure 13 value of coefficient of correlation r is 0.76. so it can be said that monthly total of predicted rainfall is more superior than annually calculated rainfall.

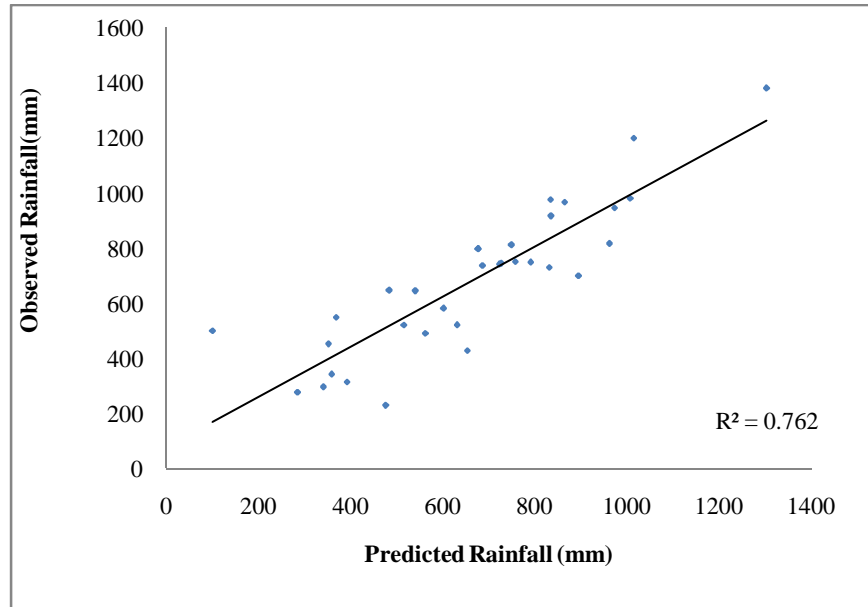


Figure 12: Comparison of annually observed and monthly total predicted Rainfall

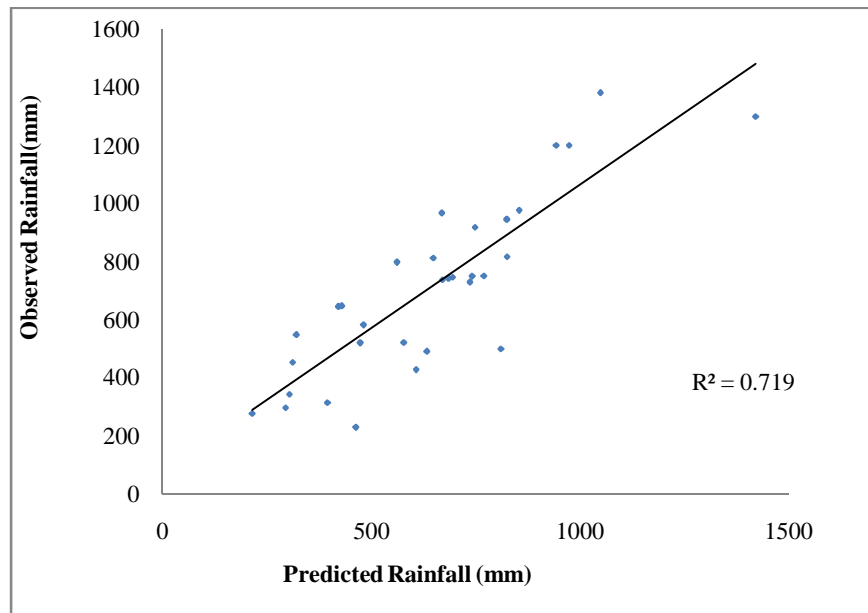


Figure 13: Comparison of annually observed and yearly predicted rainfall.

6. CONCLUSION

A rainfall model for all the district of Rajasthan is developed based on first order Markov chain process. Both monthly and yearly predicted rainfall are compared with actual rainfall for one year period. It has been observed that the model is doing a reasonably good job of prediction of rainfall. Districts like Alwar, Banswara, Barmer, Bharatpur, Bundi, Dholpur, Jaisalmer, Jalore, Jhalawar, Pali, Sawaimadhopur, Sirohi and Tonk shows very effective results. Monthly predicted data are generally needed in the simulation of water resources systems, and in the estimation of water yield from large catchments. Knowing monthly rainfall in advance helps the farmers to cultivate their crops according to water requirements.

This method is also applied for daily forecasting of rainfall but result are not feasible.

Stochastic techniques are increasing their popularity in hydrological design and analysis. These techniques are powerful tools which are available to current engineers. This study is an attempt to use stochastic technique to predict the amount of rainfall for all the districts of Rajasthan which can be hope fully be used in future hydrological studies in the area.

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